6. Saxon Bowl

Gymnázium, Plzeň, Mikulášsské nám. 23

Contents

1 Problem Statement

A bowl with a hole in its base will sink when placed in water. The Saxons used this device for timing purposes. Investigate the parameters that determine the time of sinking.

2 Introduction

2.1 History of timing devices

The first device used to measure time was a sundial. However, because sundials needed light in order to work they couldn't be used during cloudy days or night. It wasn't long until the first two different methods were introduced. The first using the movement of stars and the second using water flowing through a little hole.

The oldest water clocks come from Egypt and date back to 1500 BC[1]. The first water clocks were used more as timers than real clocks — for example, to measure intervals of speech in Classical Athens.

Because of a strong need for measuring longer periods and the ability to track the constantly changing length of more and more sophisticated designs were introduced. However, we won't cover them in this work.

Our aim will be to theoretically and experimentally examine on which parameters depends the sinking time of the simplest water-based clocks — a bowl with a hole in its bottom.

2.2 Bowl

Before jumping to the problem itself, we have to first clarify one important term - that being, what exactly is a bowl?

One could define bowl as any container which can store liquid, however this definition is vague and enables many different objects to become a bowl.

In order to prevent this, we will have two more requirements. First that the height of the bowl must be lower than the maximum diameter of itself (otherwise we would get a vase or a glass). Second that after turning the bowl around a fixed axis perpendicular to its base by arbitrary angle, we get the same shape. This implies that the horizontal cut of the bowl at any height is a disk and that all the centers of these lie on a common line perpendicular to those disks.

This is the definition we will use throughout the document[1]. Note that all common types of bowls, such as semi-spherical, conical, cylindrical or those with shape of conical frustum, do fulfil these conditions.

[1] If one used a weaker requirement on the rotational symmetry allowing bowls with shape of square frustum for example, all of the discussed proprieties and relations would hold except for those connected to the surface tension at the time of sinking in section 4.4

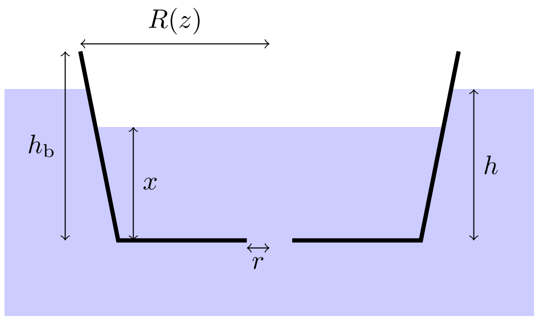

Figure 1: The bowl scheme

2.3 Parameters

We will briefly define the most important parameters and their notation.

We will denote the inside water level as x and the outside as h. Every parameters indexed with b, will refer to the bowl itself.

We will assume that the hole is circular and positioned in the center of the base. We will denote its radius r. If this was not true, then our discussions in sections 3.3 and 4.2 would have to be more complicated and would depend on the particular shape.

In correspondence to the previous subsection, we define the $R(z)$ to be the inside radius of the bowl in the given height z. Therefore integrating this function followingly should give us the total inside volume of the bowl.

$$\pi \int_0^{h_\mathrm{b}} (R(z))^2 \, dz = V$$

Also note that all derivatives are with respect to time.

3 Qualitative analysis

3.1 Applied forces

When a bowl with a hole in it's bottom is put on water, it starts sinking and the water outside starts flowing inside the bowl. As these two motions are different and different forces apply, we will discus them separately.

3.1.1 Forces on bowl

There are two main forces which act on the bowl - the gravity and the buoyant forces. These two have opposite direction, therefore one of them is cancelled and the other causes the bowl to accelerate downwards, if gravity is greater, or upwards otherwise. As the gravity force is usually higher than the buoyant (otherwise the bowl may not sink at all), the result of this is that the water level outside becomes higher than the level inside.

As gravity force is proportional to the weight of the bowl and the buoyant force increases with greater volume of air below the outside level inside the bowl, we can immediately see that as the weight increases, the time increases and as velocity of water flowing through the hole decreases, the time also decreases.

Another force that acts on the sinking bowl is drag force. The drag force is proportional to the bowl's velocity and causes it to decelerate.

3.1.2 Forces on water

Due to the motion of the bowl, water level inside is lower that that outside. This creates pressure difference outside and inside the bowl, which causes the water to start flowing inside due to the pressure-gradient force.

As this force is the difference in pressure times the area of the hole ($F = \Delta p \cdot A$), we can assume, that with increasing hole size, the water should flow in faster and as seen in previous section, the bowl should therefore sink in less time.

Note that when the hole size is large enough (i.e. $r \sim R$), the flowing through the hole is instantaneous and the whole phenomenon resembles free fall in combined environment.

Another group of forces are the forces caused by the viscosity of water and friction between the water and the base of the bowl as the water flows through the hole. These will cause the water to decelerate.

3.1.3 Surface tension

The surface tension was not discussed yet as it affects both the bowl and the water.

The surface tension first appears when the bowl starts sinking - above the hole a water bubble is created and the pressure of the water must overcome the pressure due to surface tension in order to start sinking.

Another moment when it comes to play is just before the bowl finally sinks - the surface tension creates a cap above the whole bowl which must be broken.

In both situations the sinking time is prolonged and the surface tension could also prevent the bowl from sinking.

3.2 Archimedes's principle

In the following two subsections we will discuss when and why the bowl does not sink.

The gravity force is what makes the bowl accelerate downwards, thus it must overcome the buoyant force. The gravity force however needs not to be bigger than the buoyant throughout the whole motion - only in extreme cases, that's when the water level outside equals the one inside. This can be expressed via the Archimedes's principle[2].

$$F_G \geq F_B$$

$$F_G = \rho_{\mathrm{b}} \cdot V_{\mathrm{b}} \cdot g$$

[2]Note that if the equation does not hold, the velocity at that point must be zero or tiny, therefore we neglect it.

We can take the buoyant force when it's maximal - when the bowl is almost sunk.

$$F_B = \rho \cdot V_{\mathrm{b}} \cdot g$$

$$\rho_{\mathrm{b}} \geq \rho$$

If this condition does not hold, the bowl sinks only partially.

3.3 Laplace pressure

The other reason, why the bowl may not sink, is caused by the surface tension. Before water can start flowing into the bowl, it must overcome the pressure of the surface tension. We assume, that the bubble will be semi-spherical as there the pressure is minimal. Then, the pressure the water must provide equals the Laplace pressure of the bubble.

$$\Delta P = \gamma \frac{2}{r}$$

Where γ is the surface tension for water.

In order to see whether the pressure is broken, we can imagine a situation where the water pressure is maximal – that is when the gravity and buoyant[3] forces are at equilibrium.

$$\int_0^{h_{\mathrm{e}}} (R(z))^2 \, dz = \frac{m}{\pi \rho}$$

From this, we could calculate the height of water outside at the equilibrium depending on the particular form of $R(z)$ and its antiderivative. It may also happen that there is no such equilibrium – the gravity force is always higher than the buoyant. Then we can set $h_{\mathrm{e}} = h_{\mathrm{b}}$.

We can now obtain the following requirement from the know hydro-static pressure.

$$h_{\mathrm{e}} > \frac{2\gamma}{r \rho g}$$

When this does not hold, the bowl cannot start sinking[4].

3.4 The fluid

Even though the problem statement asks for a description of a bowl placed on water, it may be beneficial to briefly discuss some differences that would occur if the bowl was placed on a different fluid.

It is intuitive to say that a bowl placed on honey or mercury would behave differently than on water. There may be more parameters causing this. The important one, we believe, are viscosity and closely connected concepts of adhesion and cohesion.

Viscosity describes how resistant is fluid to deformation. Therefore it defines how fast the fluid spreads which is something that changes how the fluid flows into a bowl. Viscosity

[3]Note that in the following formula, we neglected the buoyant force on the walls of the bowl as this one is tiny compared to the buoyant force on the air inside the bowl.

[4]It may happen that there is no equilibrium, yet the pressure is not sufficient (e.g. for an extremely tiny hole and very dense or flat bowl), then the bowl should sink from outside as if there was no hole at all. As this is a rather extreme case, we didn't observe this effect in the experiments we made and it does not capture the essence of this phenomena, we will not deal with this any further.

is characteristic quantity for any Newtonian fluid and changes with temperature. From [2] we see that viscosity of water changes noticeably when warmed. For that reason we conducted an experiment with different temperatures of water and the results are shown in experimental section (5.8).

Because of the ability of water molecules to form hydrogen bonds with their neighbors the cohesive forces are significant. We have already discussed effects of surface tension (3.1.3), so we will not deal with them here.

Capillary action may be another effect that could affect the sinking. Some of the hole-diameters we worked within our experiments were small enough that the capillary action may have occurred. However, the height of the hole is tiny which is in contrary to a typical capillary tube. Therefore we decided to neglect any such effects.

4 Quantitative model

As we discussed in the previous section, the motion of the bowl and the flow of the water can be described individually. Therefore we will derive separate equation for the motion of the bowl and the flow of the water and then link these together.

4.1 Bernoulli equation

First, let's examine the water, which is exactly inside the hole. At the beginning this water has some potential energy, initial velocity, which is zero and pressure acting on it - the atmospheric pressure.

The Bernoulli connects these properties in a way, that the following remains constant throughout the process.

$$\frac{1}{2}\rho v^2 + p + H\rho g = \text{const.}$$

Or alternatively taking the velocity to be zero at the very beginning and after subtracting atmospheric pressure from every pressure.

$$\frac{1}{2}\rho v^2 + p + \Delta H\rho g = 0$$

We can express the second and the third member in the equation in terms of x and h.

$$\frac{1}{2}\rho v^2 + x\rho g - h\rho g = 0$$

$$v = \sqrt{2|h - x|\rho g}$$

We obtained the velocity of the water inside the hole, however the Bernoulli equation requires that there must only be laminar flow and no turbulent. Therefore we must prove that the turbulent flow does not occur inside the hole.

This can be seen from the Reynolds number, which can be calculated followingly:

$$\mathrm{Re} = \frac{vr}{\nu}$$

For the bowls we used, the Reynolds number was at most 1, from which we can conclude that the turbulent flow didn't occur there and also wouldn't occur for a reasonable hole sizes, as the Reynolds number must be around 2000 for the turbulent flow to occur.

We can use the Continuity equation to link the velocity inside hole to the change in water level.

$$dV = \pi r^2 \cdot c \cdot dt$$

$$r^2 \cdot v \cdot dt = (R(x))^2 dx$$

$$\dot{x} = \frac{r^2}{(R(x))^2}\sqrt{2 \cdot |h - x| \cdot g}$$

4.2 Flow through the orifice

The equation derived in the previous subsection does not account with friction and viscous forces which play a significant role when the water flows through sharp orifice.

If we take these into account, the flow becomes lower than the estimated. The Discharge coefficient puts these two together.

$$C_{\mathrm{d}} = \frac{Q_{\mathrm{exp}}}{Q_{\mathrm{theo}}}$$

Where Q_{exp} is the real discharge and Q_{theo} is the theoretically calculated discharge. As the area stays the same, we could write:

$$C_{\mathrm{d}} = \frac{v_{\mathrm{exp}}}{v_{\mathrm{theo}}}$$

Therefore our equation if we take these forces into the account would look as follows:

$$\dot{x} = C_{\mathrm{d}}\frac{r^2}{(R(x))^2}\sqrt{2 \cdot |h - x| \cdot g}$$

This is the differential equation describing motion of the water.

We use $C_d = 0.61$ which is a standard value for an orifice [4]. The value could vary slightly based on different size of the hole and velocity of water but those changes should be insignificant and should not change the prediction.

4.3 Laws of motion

We now have to examine the behavior of the bowl. As was stated in qualitative analysis, there are three forces acting on the bowl - the gravity, the buoyant and the drag force. The resulting force is the difference of these.

$$F = F_{\mathrm{G}} - F_{\mathrm{B}} - F_{\mathrm{D}}$$

Where the gravity force equals:

$$F_{\mathrm{G}} = mg$$

The buoyant force is the sum of the buoyant force acting on the bowl[5] and the force acting on the air inside the bowl.

$$F_{\mathrm{B}} = \pi \int_x^h (R(z))^2 \, dz \cdot \rho \cdot g + m \cdot g \cdot \frac{\rho}{\rho_{\mathrm{b}}} \frac{h}{h_{\mathrm{b}}}$$

The drag force can be calculated from the velocity of the bowl.

$$F_{\mathrm{D}} = \frac{1}{2} C_{\mathrm{D}} \cdot \rho \cdot S_{\mathrm{D}} \cdot \dot{h}^2$$

Where S_{D} is the surface affected by drag and C_{D} is the drag coefficient, which depends on the bowl's shape. It might also happen during the sinking, that due to great difference between levels outside the bowl starts moving upwards. Then the drag force has opposite direction and the coefficient might also change at that moment.

Putting these together we obtain the resultant force.

$$F = m \cdot g - \pi \int_x^h (R(z))^2 \, dz \cdot \rho \cdot g - \frac{g \cdot \rho \cdot h}{\rho_{\mathrm{b}} \cdot h_{\mathrm{b}}} - \frac{1}{2} C \cdot \rho \cdot S_{\mathrm{D}} \cdot \dot{h}^2$$

We can use the Laws of motion to link this force with bowl's acceleration.

$$\ddot{h} = \frac{F}{m}$$

$$\ddot{h} = g - \frac{\pi \int_x^h (R(z))^2 \, dz \cdot \rho \cdot g}{m} - \frac{g \cdot \rho \cdot h}{\rho_{\mathrm{b}} \cdot h_{\mathrm{b}}} - \frac{C \cdot \rho \cdot S_{\mathrm{D}} \cdot \dot{h}^2}{2m}$$

Thus, we obtained differential equation describing the motion of the bowl.

4.4 Sinking

The last thing that needs to be done is to determine when the bowl sinks.

When the level of water from outside reaches the height of the bowl, the water flows into the bowl from the top and in a moment, the bowl sinks. However, first it must break the pressure caused by surface tension at the top of the bowl.

We can once again calculate this using the Laplace pressure.

$$\Delta p = \frac{2\gamma}{R(h_{\mathrm{b}})}$$

As the bowl dives deeper, the hydro-static pressure from the additional height increases and when this pressure equals the Laplace pressure, the water flows in and the bowl sinks.

$$h = \frac{2\gamma}{R(h_{\mathrm{b}})\rho g}$$

This height will act as an imaginary increase to the height of the bowl. Therefore we can write for the height of sinking:

$$h_{\mathrm{s}} = h_{\mathrm{b}} + \frac{2\gamma}{R(h_{\mathrm{b}})\rho g}$$

[5] We assumed the walls of bowl have constant thickness. If this was not true, than the second member would have to be more general as follows. $F_{\mathrm{B_w}} = m \cdot g \cdot \frac{\rho}{\rho_{\mathrm{b}}} \frac{V_{\mathrm{w}}(h)}{V_{\mathrm{w}}(h_{\mathrm{b}})}$ Assuming $V_{\mathrm{w}}(z)$ describes the volume of bowl's walls up to the given height z.

4.5 Summary of the model

We derived two differential equations. One for the bowl and one for the water flowing inside.

$$\dot{x} = C_{\mathrm{d}} \frac{r^2}{(R(x))^2} \sqrt{2 \cdot |h - x| \cdot g}$$

$$\ddot{h} = g - \frac{\pi \int_x^h (R(z))^2 \, dz \cdot \rho \cdot g}{m} - \frac{g \cdot \rho \cdot h}{\rho_{\mathrm{b}} \cdot h_{\mathrm{b}}} - \frac{C \cdot \rho \cdot S_{\mathrm{D}} \cdot \dot{h}^2}{2m}$$

As solving these analytically would be very difficult or even impossible, we decided to solve them numerically.

We implemented the iterated Backward Euler method (as it is less prone to errors than the Forward Euler method) in programming language Python.

The initial conditions were all set to zero ($x(0) = 0$, $h(0) = 0$ and $\dot{h}(0) = 0$) and the computation stopped whenever $h = h_{\mathrm{b}} + \dfrac{2\gamma}{R(h_{\mathrm{b}})\rho g}$.

5 Experiments

All the measured data that are not present in this section can be found in the Appendix.

5.1 Used bowls

Throughout the unexpected quarantine that was set due to the COVID-19 in the Czech Republic, our ability to use 3D printed bowls was limited. Therefore we decided to use two different types of bowls for experiments.

The first bowl was 3D printed from polyactic acid (PLA) and had cylindrical shape. PLA has density $\rho = 1241 \mathrm{kg} \cdot \mathrm{m}^{-3}$ [5] and so it sinks when placed on water.

Parameters of the cylindrical bowl:

$$h = (60 \pm 0.02) \mathrm{mm}$$

$$d = (5 \pm 0.02) \mathrm{mm}$$

where h is the height and d is the diameter of the hole, as seen on Figure 2. We do not provide diameter and mass on purpose. This type of bowl was used for measuring the dependency on volume for which we printed many bowls with varying diameter and consequently a different mass.

The second was a cup from Cottage cheese. This cup had walls that diverted from the centre and thus the cup had the shape of a conical frustum. However, our model can count for this different shape and predict the bowl's behaviour. The disadvantage is that the bowl was made from polypropylene which has a density lower than water: $\rho = 850 \mathrm{kg/m}^3$[6]. This we solved by putting play-dough at the bottom of the cup consequently increasing the overall density (this is discussed in detail in the following two sections).

Parameters of the conical frustum bowl:

$$d_1 = (61.90 \pm 0.02)\text{mm}$$

$$d_2 = (84.30 \pm 0.02)\text{mm}$$

$$h = (42.80 \pm 0.02)\text{mm}$$

$$m = (6.0 \pm 0.5)\text{g}$$

where d_1 is the diameter of the bottom, d_2 the diameter of the top and h the height as show on the following schema:

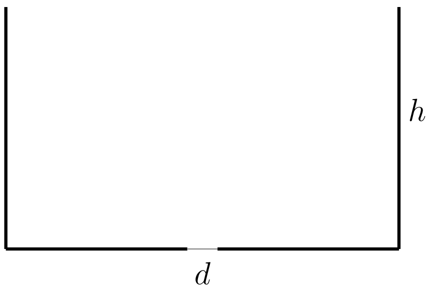

Figure 2: Cylindrical frustum bowl schema

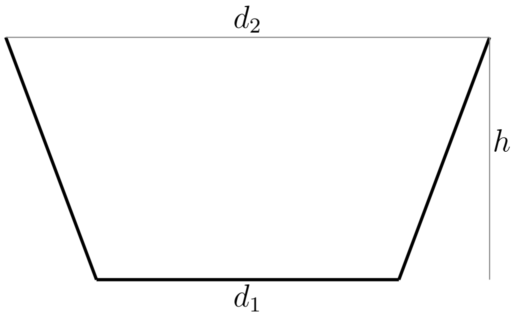

Figure 3: Conical frustum bowl schema

5.1.1 Increasing density

In the section 3.2, we showed that whether a bowl sinks depends on its density. Because the conical frustum bowl had lower density than water, we decided to increase its mass with play-dough.

We tried two ways of placing the play-dough. In the first case, we tried to put it inside. This, however, resulted in instability and a change in inner volume. Because of that, we settled to placing the play-dough at the bottom (proposed by [7]) as shown in the following two pictures. This method did not change the volume and resulted in a lower centre of mass, therefore making the bowl more stable on water.

Figure 4: Conical frustum bowl front

Figure 5: Conical frustum bowl bottom

5.1.2 Play-dough

We experimented with two types of play-dough - Play-Doh and KOH-I-NOOR. The first softened and changed its structure when immersed in water. However, we did not observe any visible signs that water would alter the structure of the second type. As a consequence, we used only the KOH-I-NOOR play-dough.

From Archimedes principle, we were able to estimate its density as: $\rho = 1767\text{kg/m}^3$. From the formula for propagation of uncertainty we calculated standard deviation as:

$$s_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 s_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 s_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 s_z^2 + ...}$$

$$s = \sqrt{\left(\frac{1}{9 \cdot 10^{-3}}\right)^2 10^{-6} + \left(-\frac{0.159}{8.1 \cdot 10^{-5}}\right)^2 2.5 \cdot 10^{-12}}$$

$$s \approx \pm\frac{1}{9}\text{kg/m}^3$$

And for density of play-dough we therefore got:

$$\rho = 1767 \pm \frac{1}{9}\text{kg/m}^3$$

5.2 Experimental approach

With both types of bowl, we begun by placing the bowl on the water surface as gently and evenly as possible. When the bowl floated on the water surface and hadn't began sinking, we applied a small initial push in downwards direction needed to initiate sinking. This was happening just with the smaller diameter holes where a spherical cup formed inside the hole (as discussed in 3.3). However, this small initial push should not affect the sinking time significantly, as it serves only to overcome the surface tension.

We also observed the effects of wetting, when the dry bowl sank slightly longer than the same bowl after it was wetted. To prevent it from affecting the measurements, all of the experiments were made with a wet bowl.

5.2.1 Experiments with cylindrical bowl

We used the cylindrical bowl for measuring the dependency on volume (5.7) and the effect of viscosity (5.8).

In 5.7 we altered the diameter while the height remained the same. Varying height would result in instability of the bowl during the initial part of the experiment thus we see varying diameter as less error-prone method.

5.2.2 Experiments with conical frustum bowl

We used the conical frustum bowl for measuring the dependency on radius (5.5) and dependency on density (5.6).

When experimenting with the conical frustum bowl with 1.5 mm diameter hole, we noticed an air-bubble forming under the bowl bottom due to the play-dough that was preventing the air from escaping. This bubble significantly affected some of the experiments with small hole diameters by partially blocking the hole and therefore slowing the water flow into the bowl. With bigger holes, the bubble was able to escape through and didn't affect the experiment much. However, to prevent both of those scenarios, we made a narrow hole at the play-dough-bowl conjunction to create an escape-route for the air and put a mirror on the bottom of the tank, as seen on the image bellow, so we could find out whether a bubble had formed. Any experiment when the bubble occurred was interrupted and repeated.

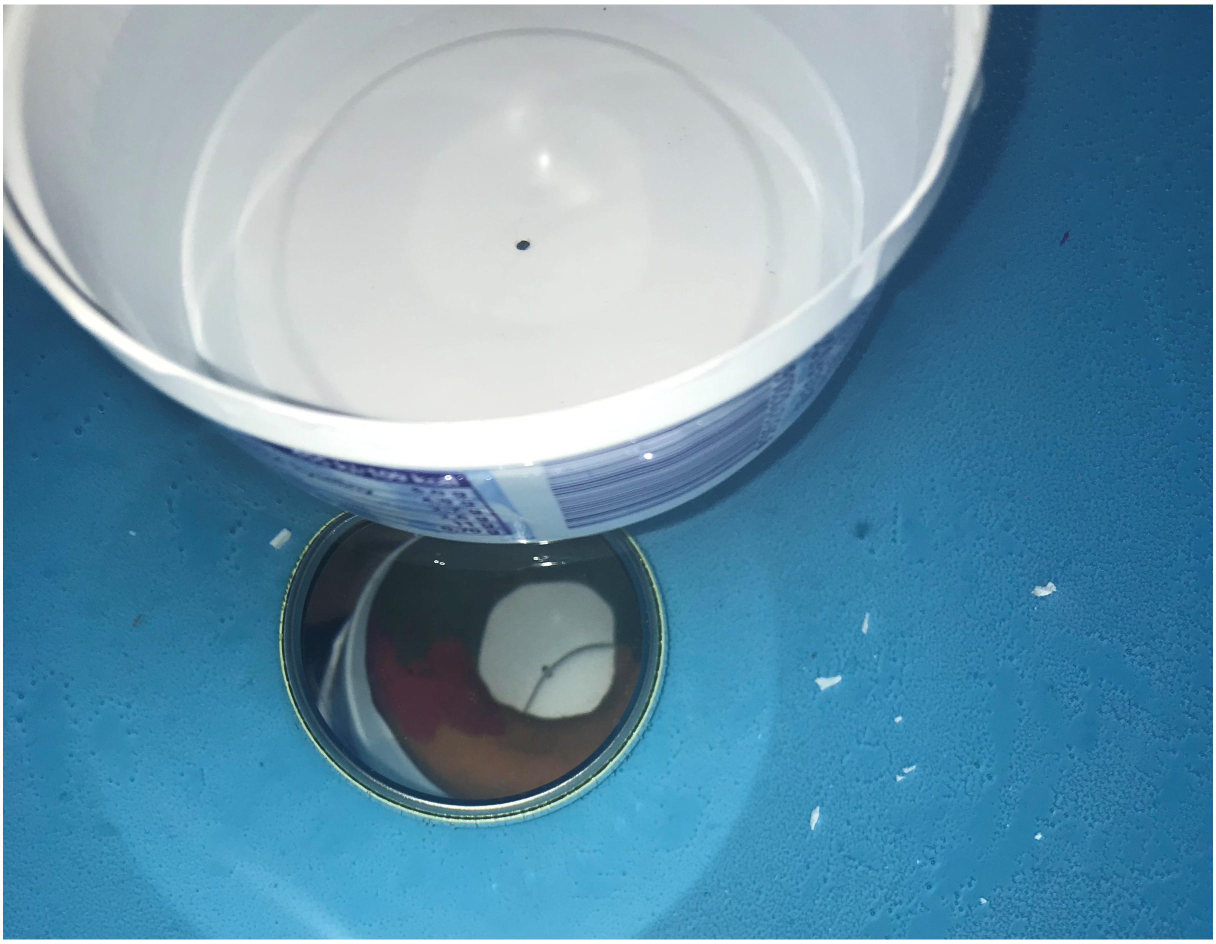

Figure 6: Air bubble under the bowl

5.3 Conditions in laboratory

We conducted our experiments in two different places. Measurements 5.5 and 5.6 were conducted in a laboratory with: $t = 22°C$, $p = 1007$hPa and $\Phi = 71\%$ where t is the temperature, p the pressure and Φ the relative humidity and experiments 5.7 and 5.8 in house with: $t = 23.2°C$ and $\Phi = 29\%$ and an unknown pressure which we believe to be comparable to the one in the lab.

5.4 Reaction time

In all of our experiments we were tracking the time with watches which were stopped when the bowl got under the surface of water. Although the watch were always operated by a human experimenter we believe the reaction time can be neglected. Here we present two reasons why.

The time was started and stopped by the same person. If the delay was added to the time at the beginning the similar effect must have happened at the end. Those two delays will have different lengths, however, we argue that they partially cancel out and make the effect of reaction time smaller.

We also measured our own reaction time [6]. The person who conducted vast majority of experiments hat an average reaction time $t_r = 309\text{ms}$. Compared to standard deviations of majority of our experiments this number is small. Therefore, we have decided to neglect the effect of reaction time in our findings.

5.5 Dependency on the orifice diameter

In this experiment, we used bowls with the shape of the conical frustum (5.1). To test our hypothesis about the diameter, we drilled circular holes of different radii to bottoms of the bowls (always in the middle) and tracked time needed for the bowl to sink. Each bowl was loaded with 60g of play-dough which gives as overall density: $\rho = 1611\text{kg/m}^3$. We concluded 7 measurements for each diameter.

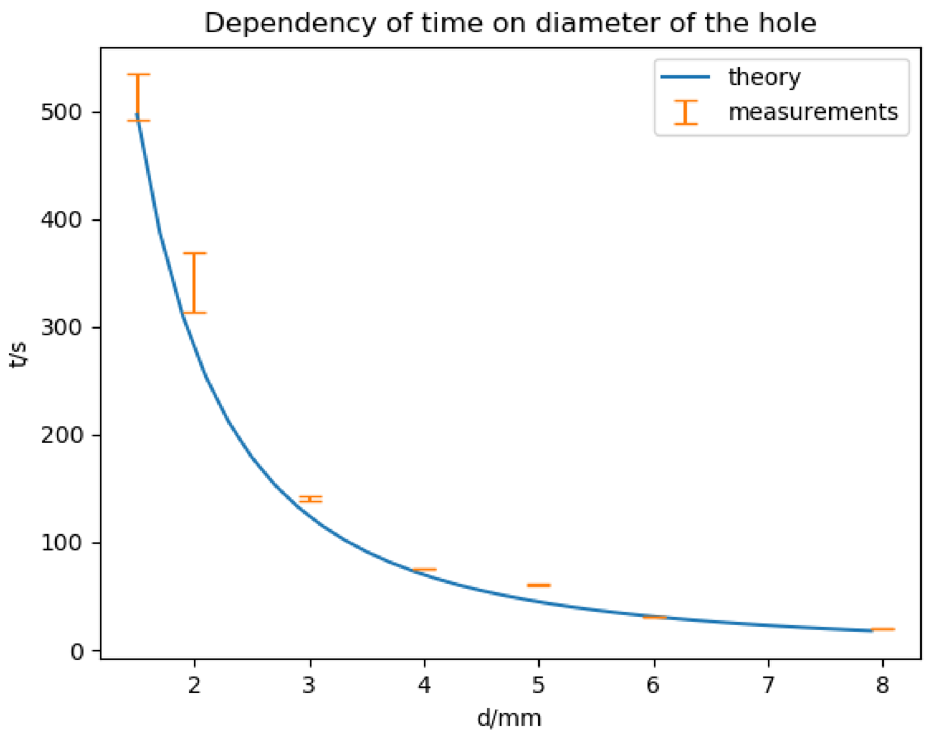

Figure 7: Dependency of time on diameter of the orifice

[6]We used following web page to do so: `https://www.humanbenchmark.com/tests/reactiontime`

The measured values match very closely with the values computed from the simulation, although they appear to be a bit higher than those predicted by our model.

5.6 Dependency on density

We do not explicitly discuss the dependency on density in our theory, although section 4.5 implies that there should be a relation between time and density.

For that reason we experimentally examined this relation. We used the conical frustum bowl with a hole of diameter $d = 4$mm. Throughout the experiment, we were increasing the weight of play-dough (from 60 g to 100 g) added to the bottom, thus increasing overall density.

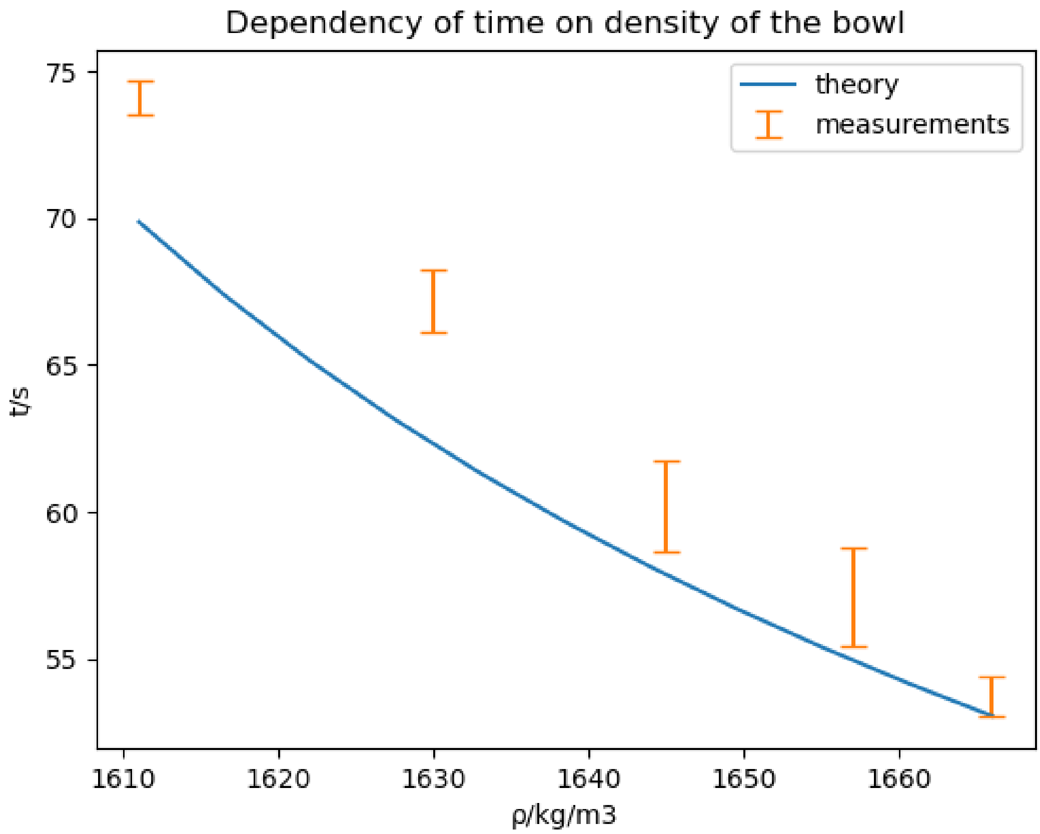

Figure 8: Dependency of time on density

We can see that the simulation here is steadily under the measured values, although they are following a similar trend. We explain this by an error in determining the density of the bowl (there are different types of polypropylene) and play-dough for which the method from Archimedes principle might not be perfectly precise. This could be also showing in Figure 7, a little, where the measured values are also slightly higher than those predicted. However, in both of those experiment, we can see that the trend given by our computation is clearly followed - this also proves our hypothesis.

5.7 Dependency on volume

Cylindrical bowl was used in this experiment. We varied the diameter of the bottom, thus increasing volume. The diameter ranged from 6 to 10 cm and the height was fixed at 6 cm. The measurements for each diameter were repeated 5 times. With the biggest diameter, the water flowed to the side and the bowl started tilting significantly. This could theoretically change the height of the hole and therefore affect the results. Because of that, we did not increase the diameter even more.

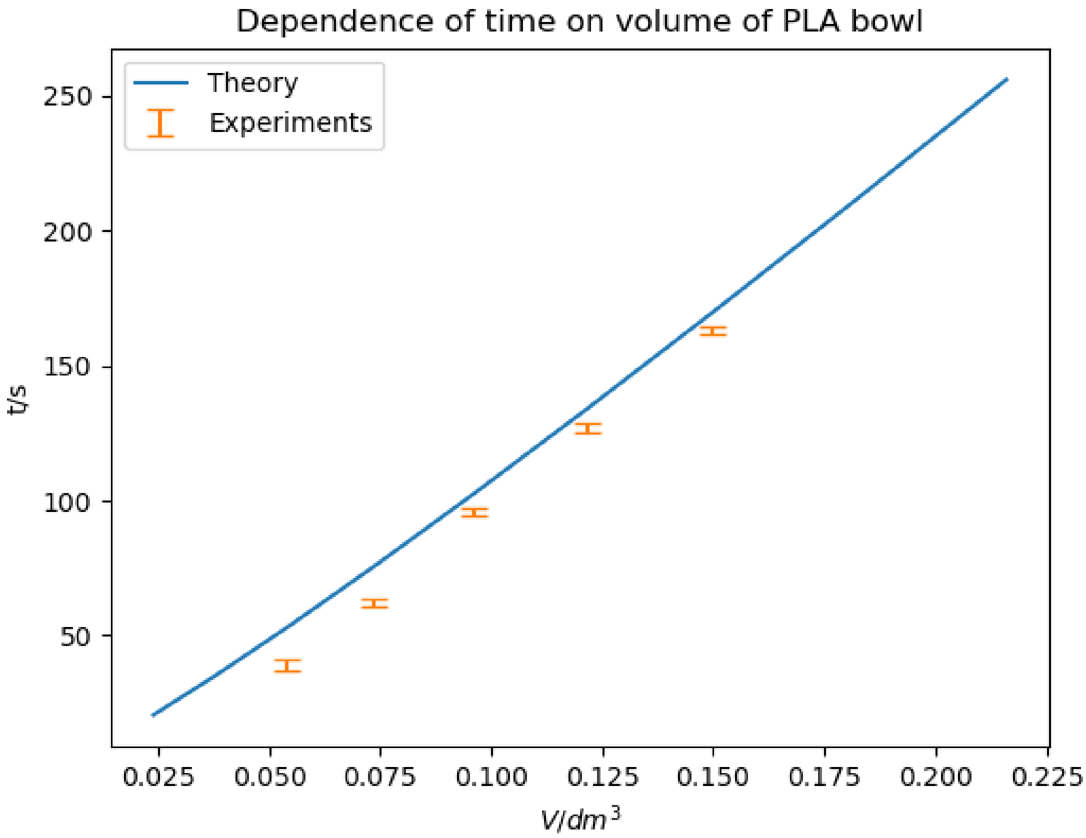

Figure 9: Dependency of time on volume

The data, as seen on the graph in Figure 9, show that there is a linear dependency between the volume and time needed for sinking. We can see that the measured values are slightly bellow the predicted values but following a very similar trend. This can be explained by the fact that our model works with several coefficients that were not possible to experimentally verify and which may vary from those used in the simulation.

5.8 Effect of viscosity

We experimented with two temperatures (4 measurements each) of water $T_1 = 20 \pm 0.1°C$ and $T_2 = 60.3 \pm 0.1°C$. We used 3D printed bowls for this experiment even though the PLA has lower melting point than polypropylene. Those could not be used because of low melting point of our play-dough. It was not possible to sustain the temperature during the whole sinking. However, we monitored the temperature during the whole process and we found that in both examples the difference between the temperature at the beginning and at the end was smaller than $2°C$. Measured results (where ν is kinematic viscosity):

$\frac{T}{°C}$	20.0 ± 0.1	60.3 ± 0.1
$\frac{t_{\text{avg}}}{s}$	21.0 ± 0.4	20.9 ± 0.5
$\frac{\nu}{m^2 \cdot \frac{1}{s} \cdot 10^6}$	1.004	0.475

We conducted 4 measurements for two temperatures and can conclude that the effect of changing viscosity of water due to temperature is insignificant.

6 Conclusion

In our work, we presented a thorough description of the Saxon bowl phenomena. We discussed many different parameters that affect the sinking time of a bowl, after that a comprehensive numerical model was created. This model was later verified experimentally. From our experiments, it is possible to see that sometimes measurements do not fit exactly into the theory. This may be due to the fact that our model uses several coefficients that can greatly affect the outcome of simulation and can not be always precisely calculated. Nevertheless, our model gives a complex description of the phenomena and as experiments showed it can predict the trend well in a broad amount of different initial setups.

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[7] Canadian Young Physicists' Tournament, IYPT 2020 Problem 6 Saxon Bowl Demonstration, www.youtube.com/watch?v=rr1r9yImb_M

A Dependency on diameter - data

7 measurements were conducted for each size of the drill bit.

$\frac{d}{mm}$	1.50 ± 0.02	2.00 ± 0.02	3.00 ± 0.02	4.00 ± 0.02	5.00 ± 0.05	6.00 ± 0.05	8.00 ± 0.05
$\frac{t_1}{s}$	549.1	298.2	139.1	74.8	60.6	31.0	19.7
$\frac{t_2}{s}$	501.2	305.3	138.9	74.6	61.3	31.3	19.7
$\frac{t_3}{s}$	532.2	348.0	141.7	74.9	60.4	30.5	19.4
$\frac{t_4}{s}$	497.8	358.6	143.2	75.1	60.8	30.7	20.2
$\frac{t_5}{s}$	491.4	361.7	142.4	75.3	62.2	30.9	20.4
$\frac{t_5}{s}$	502.6	365.4	140.2	74.6	61.7	31.2	19.8
$\frac{t_5}{s}$	518.1	139.9	139.5	75.2	60.4	30.4	19.6
$\frac{t_{\text{avg}}}{s}$	513.2 ± 19.4	341.0 ± 25.5	140.7 ± 1.6	74.9 ± 0.3	61.1 ± 0.6	30.9 ± 0.3	19.8 ± 0.3
$\frac{t_{\text{theo}}}{s}$	496.6	279.4	124.2	69.9	44.0	31.1	17.5

B Dependency on density - data

5 measurements were conducted for each density. m_{pd} is the mass of play-dough, t_{avg} average value and t_{theo} calculated value from theory.

$\frac{\rho}{\frac{kg}{m^3}}$	1611	1630	1645	1657	1666
$\frac{m_{pd}}{g}$	60.00 ± 0.05	70.00 ± 0.05	80.00 ± 0.05	90.00 ± 0.05	100.00 ± 0.05
$\frac{t_1}{s}$	73.8	68.6	62.3	54.9	53.5
$\frac{t_2}{s}$	73.9	65.7	60.7	59.3	52.7
$\frac{t_3}{s}$	74.6	67.4	60.3	56.2	53.5
$\frac{t_4}{s}$	74.9	67.6	59.8	57.5	54.4
$\frac{t_5}{s}$	73.5	66.9	58.0	57.7	54.3
$\frac{t_{\text{avg}}}{s}$	74.1 ± 0.5	67.2 ± 0.1	60.2 ± 1.4	57.1 ± 1.5	53.7 ± 0.6
$\frac{t_{\text{theo}}}{s}$	69.5	64.9	61.1	57.9	55.2

C Dependence on volume - data

The data for different initial diameters of cylindrical bowl. Diameter of the hole was $d_h = (5.00 \pm 0.02)$mm during the whole experiment. d_b is the diameter of the bottom, t_{avg} average time and t_{theo} calculated value from theory.:

$\frac{d_b}{cm}$	3.00 ± 0.02	3.50 ± 0.02	4.00 ± 0.02	4.50 ± 0.02	5.00 ± 0.02
$\frac{t_1}{s}$	37	62	95	125	165
$\frac{t_2}{s}$	37	63	94	129	164
$\frac{t_3}{s}$	42	62	95	130	163
$\frac{t_4}{s}$	40	64	98	128	161
$\frac{t_5}{s}$	39	60	96	127	162
$\frac{t_{\text{avg}}}{s}$	39 ± 7	62 ± 5	96 ± 5	128 ± 7	163 ± 4
$\frac{t_{\text{theo}}}{s}$	58	75	102	134	170

D Effect of viscosity

The data for experiment with viscosity of water and its effect on sinking:

$\frac{T}{°C}$	20.0 ± 0.1	60.3 ± 0.1
$\frac{t_1}{s}$	21.1	20.7
$\frac{t_2}{s}$	20.9	21.1
$\frac{t_3}{s}$	21.14	20.9
$\frac{t_4}{s}$	21.2	21.0
$\frac{t_{\text{avg}}}{s}$	21.0 ± 0.4	20.9 ± 0.5
$\frac{\nu}{m^2 \cdot \frac{1}{s} \cdot 10^6}$	1.004	0.475